Study

Power System 2

Performance of Long T.L

$$\boxed{\theta = \sqrt{zy}}$$ Propagation factor

$$\theta = \alpha(attemuation\ constant) + j\ \beta\ (\ phase\ constant\ )$$

$$Z_0 = \sqrt{\frac{z}{y}}\ \ the\ charactestic\ impedence\ of\ the\ line$$

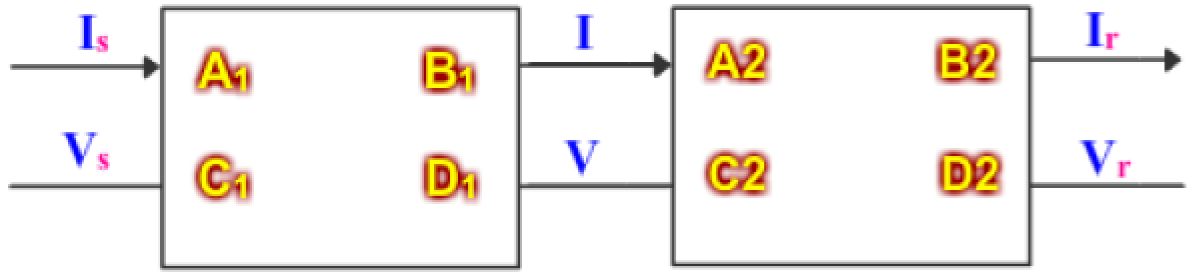
$$V_S = A\ V_r + B\ I_r$$
$$I_s = C\ V_r + D\ I_r$$

$$A = D = \cosh(\theta) = 1 + \frac{ZY}{2} \quad B = \frac{Z\sinh\theta}{\theta} = Z\left(1 + \frac{ZY}{6}\right) \quad C = \frac{Y\sinh\theta}{\theta} = Y\left(1 + \frac{ZY}{6}\right)$$

Nominal T	Nominal π
$A = D = 1 + \frac{Z'Y'}{2} = \cosh\theta$ $B = Z'\left(1 + \frac{Z'Y'}{4}\right) = \frac{Z\sinh\theta}{\theta}$ $C = Y' = \frac{Y\sinh\theta}{\theta}$ Where $Y' = \frac{Y\sinh\theta}{\theta}$ $Z' = Z\frac{\left(\tanh\left(\frac{\theta}{2}\right)\right)}{\frac{\theta}{2}} = Z(1 - \frac{ZY}{12})$	$A = D = 1 + \frac{Z'Y'}{2} = \cosh\theta$ $B = Z' = \frac{Z\sinh\theta}{\theta}$ $C = Y'\left(1 + \frac{Z'Y'}{4}\right) = \frac{Y\sinh\theta}{\theta}$ Where $Z' = \frac{Z\sinh\theta}{\theta}$ $Y' = Y\frac{\left(\tanh\left(\frac{\theta}{2}\right)\right)}{\frac{\theta}{2}} = Y(1 - \frac{ZY}{12})$

General Constant of

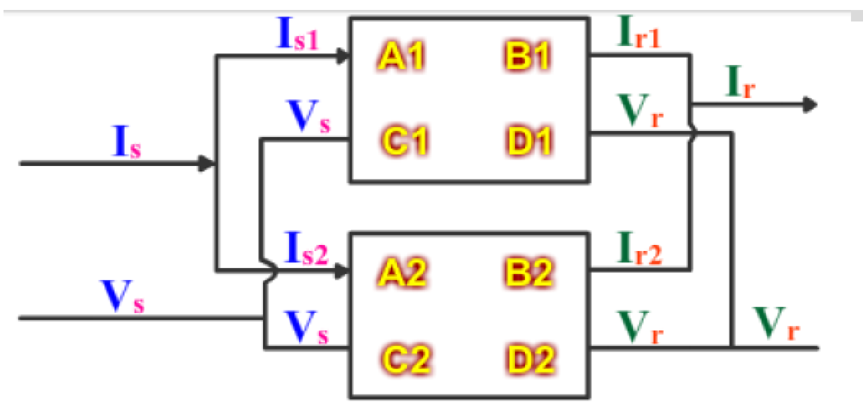
1- Cascade T.L



$$A = A_1A_2 + B_1C_2 \qquad B = A_1B_2 + B_1D_2$$

$$C = C_1A_2 + D_1C_2 \qquad D = C_1B_2 + D_1D_2$$

2- Parallel T.L

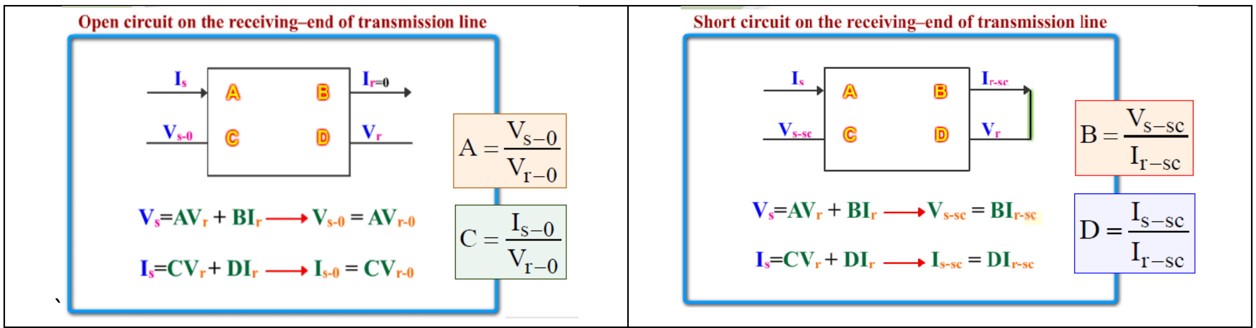


$$A = \frac{A_1B_2 + B_1A_2}{B_1 + B_2} \qquad B = \frac{B_1B_2}{B_1 + B_2}$$

$$C = C_1 + C_2 + \frac{(A_2 - A_1)(D_1 - D_2)}{B_1 + B_2}$$

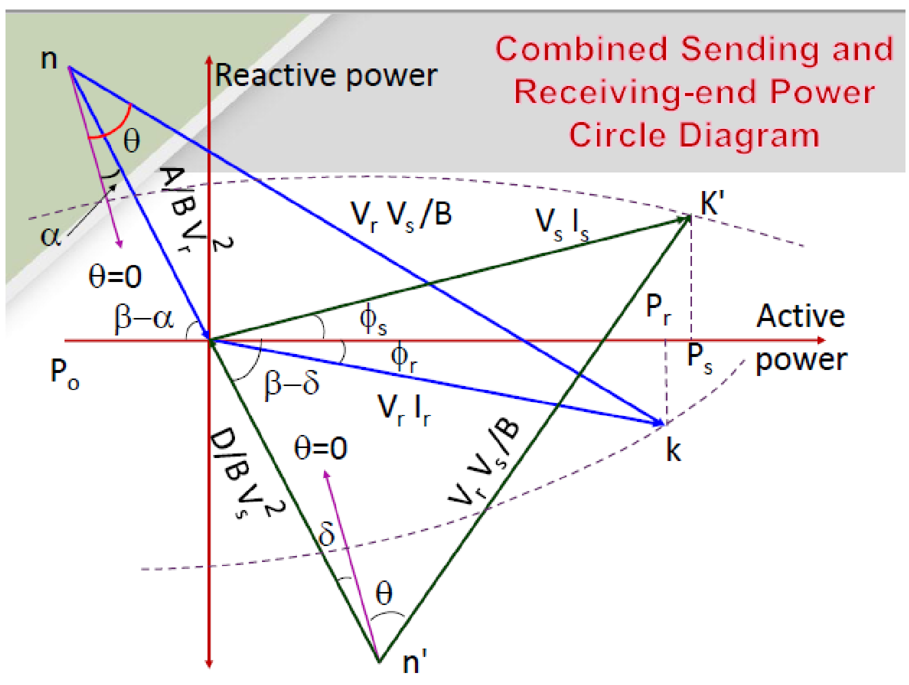
$$D = \frac{B_1D_2 + B_2D_1}{B_1 + B_2}$$

Experimental Determination of the ABCD

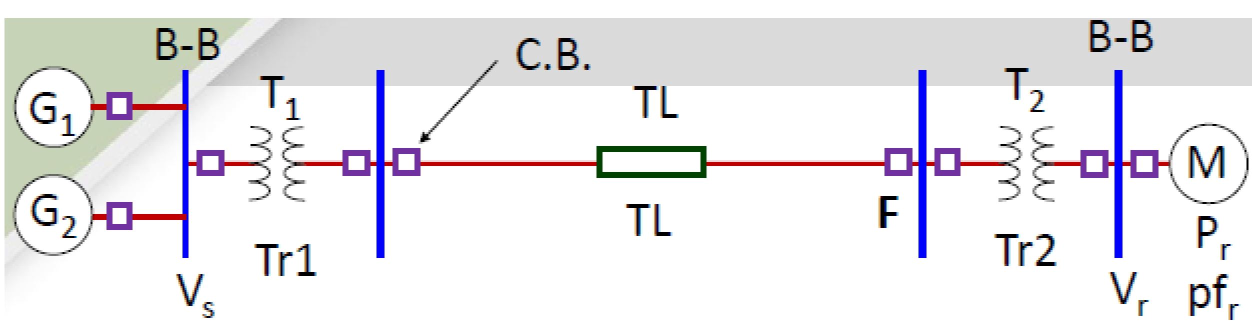


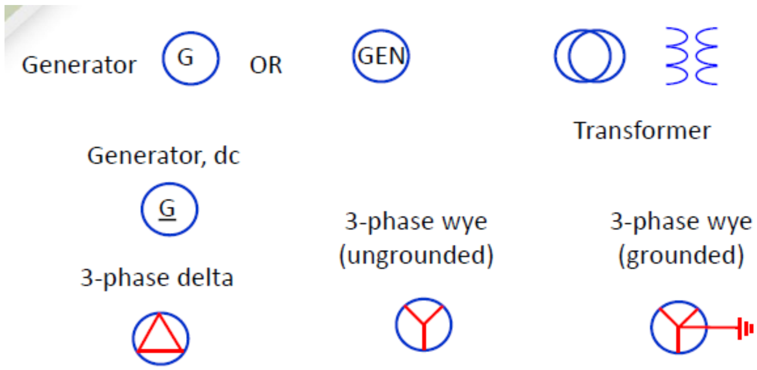
POWER CIRCLE DIAGRAM

Just know the steps from the drawing not by writing them and solve by hands



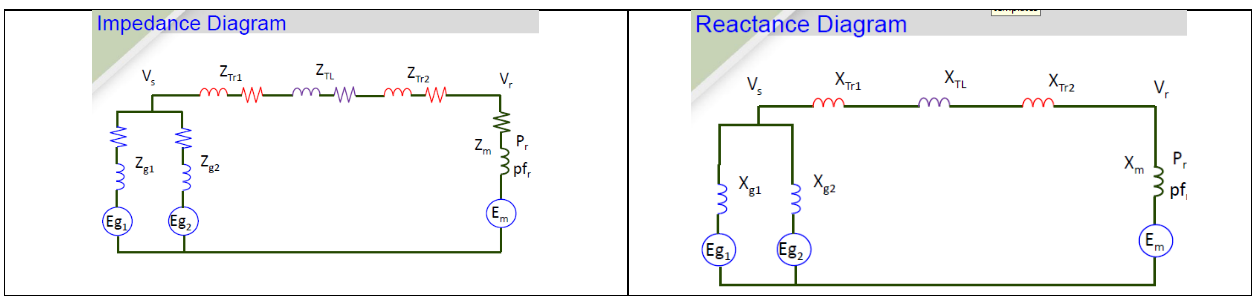
Single-line diagram





Impedance Diagram

- The resistance of bus bars is neglected
- The magnetizing impedances of transformers are neglected
- The resistances of all components may be neglected to obtain the reactance diagram since the resistance is much smaller than the reactance



PER UNIT QUANTITIES

$$\boldsymbol{Per - unit\ value = \frac{Actual\ Value}{Base\ Value}}$$

Per unit is **<u>dimensionless</u>** cuz actual value and base value have the same unit

Base value is always **a real number**

The **angle** of per unit value is the **same** as the actual value

$$\frac{S\angle\theta}{S_{base}} = \frac{V\angle\alpha \cdot I\angle -\beta}{S_{base}}$$

$$I_{base} = \frac{S_{base}}{V_{base}}$$

$$S_{pu}\angle\theta = \left(\frac{V\angle\alpha}{V_{base}}\right)\left(\frac{I\angle-\beta}{I_{base}}\right)$$

$$S\angle\theta = V\angle\alpha \cdot I\angle-\beta$$

$$S_{pu} = V_{pu}\angle\alpha\left(I_{pu}\angle-\beta\right) = \quad S_{pu} = V_{pu} I^{*}{}_{pu}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

$$Z_{pu} = \frac{Z}{Z_{base}} = \frac{R+jX}{Z_{base}} = \left(\frac{R}{Z_{base}}\right) + j\left(\frac{X}{Z_{base}}\right)$$

$$Z_{pu} = R_{pu} + j X_{pu} \qquad Z_{base} = R_{base} = X_{base}$$

$$S_{base} = P_{base} = Q_{base}$$

Impact of transformer

$$V_{2-base} = \frac{N_2}{N_1} V_{1-base} \qquad \frac{V_1}{V_{1-base}} = \frac{\frac{N_1}{N_2} V_2}{\frac{N_1}{N_2} V_{2-base}} \qquad V_{1-pu} = V_{2-pu}$$

$$S_{1-base} = S_{2-base} = S_{base} \quad i_{base} = \frac{S_{base}}{V_{base}} \quad \boxed{I_{2-base} = \frac{N_1}{N_2} I_{1-base}}$$

Note :

عند وجود 3 سينجل فيز ترانسفورمر يتم استخدامهم ك 3 فيز ، يتم ضرب الجهد في ناحيه ال y في $\sqrt{3}$ بينما في ناحيه الدلتا تبقى كما هي

Steps of converting **a single phase** system to Per Unit

- Choose MVA base **MVA$_{base}$** for the whole system

•Choose a kilo voltage base **kV$_{base}$** for one section

•Calculate the voltage base in other sections in the network using transformation ratio of transformers

Calculate the impedance base and current base using MVAbase and kVbase in each section•

Steps of converting a **three phase** system to Per Unit

•Choose an apparent power base "**VA**3-**base**" for the three phase for all parts in the system

•Choose a line-to-line voltage base **VL-base** for one section in the circuit

Calculate the voltage base in other sections in the network using transformation ratios of transformers • considering the method of winding connection

Calculate the impedance base and current base in terms of VAbase and Vbase in each section of the • :power system using the following equations

$$I_{base} = \frac{S_{3-base}}{\sqrt{3}V_{L-base}}$$

For star connection: $Z_{base} = \frac{(V_{L-base})^2}{S_{3-base}}$

For delta connection: $Z_{base} = 3\frac{(V_{L-base})^2}{S_{3-base}}$

Changing The Base

$$X_{pu(new)} = X_{pu(old)} \frac{VA_{b(new)}}{VA_{b(old)}} \left(\frac{V_{b(old)}}{V_{b(new)}} \right)^2$$

PU System advantages

- **Simplifies** numerical **calculations**

•The **voltage** throughout the power system is normally close to **unity**

•the **parameters** of all components tend to fall into a relatively **narrow range**, making incorrect values noticeable

•Useful in **simulating** machines on analog and digital computers for steady-state and dynamic analysis

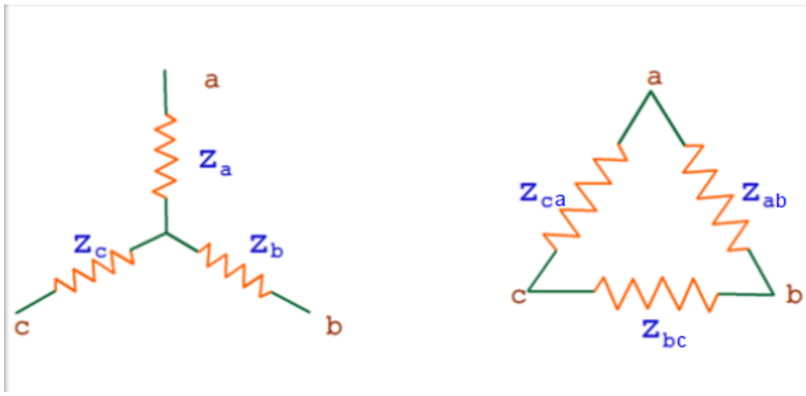
•There is **no** large **difference** between **single**-and **3-phase** systems or between line and phase voltages

•Manufacturers usually **define** the **impedance** of all equipments in **p.u**. and it can be used directly if the bases chosen are the same as the name plate rating

•The **referring** process **in** the presence of **transformers** is **eliminated** and the **p.u.** equivalent impedance of any transformer is the **same** referred to either **primary or secondary** side

Node Elimintion by Star delta

-when voltage is not important – study stability

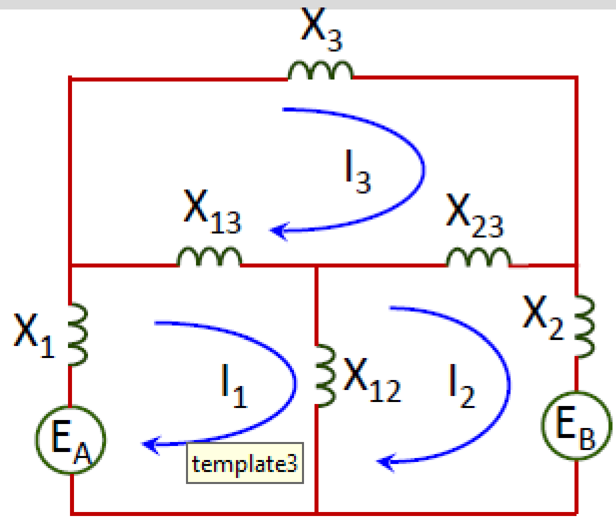


$Y\ to\ \Delta$	$\Delta\ to\ Y$
$Z_{ab} = \dfrac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$	$Z_a = \dfrac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$
$Z_{bc} = \dfrac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$	$Z_b = \dfrac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}$
$Z_{ca} = \dfrac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$	$Z_c = \dfrac{Z_{bc} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$

Loop Equation

- Assume a current in each loop
- For each loop, apply Kirchhoff's voltage law

$$\sum \text{emf} = \sum IX$$



$$E_1 = Z_1 I_1 + Z_{12}(I_1 - I_2) + Z_{13}(I_1 - I_3) = (Z_1 + Z_{12} + Z_{13})\, I_1 - Z_{12} I_2 - Z_{13} I_3$$

$$E_1 = Z_{11}\, I_1 - Z_{12} I_2 - Z_{13} I_3$$

$$E_2 = -\, Z_{21}\, I_1 + Z_{22}\, I_2 - Z_{23} I_3$$

$$E_3 = -\, Z_{31}\, I_1 - Z_{32}\, I_2 + Z_{33} I_3$$

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & -Z_{12} & -Z_{13} \\ -Z_{21} & Z_{22} & -Z_{23} \\ -Z_{31} & -Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\mathbf{E} = \mathbf{Z}_{\mathbf{loop}}\, \mathbf{I}$$

$$I = Z_{\text{loop}}^{-1} E$$

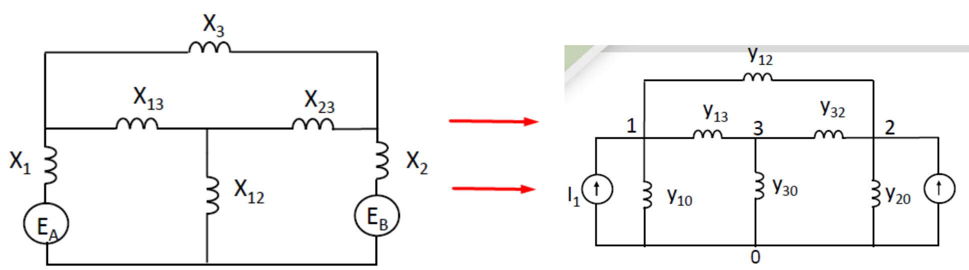
Z_{loop} is the impedance matrix, which has the following characteristics:

- It is a square matrix
- The main diagonal contains positive impedances that represent the self impedance in the loop
- The off diagonal elements are all negative and represent the mutual impedances between loops

Node Equation

•Convert the impedance representation to the admittance representation

•Convert the voltage sources to current sources



tem

Apply Kirchhoff's current law at each node

$$\boxed{\sum I = 0}$$

$$I_1 = y_{10}(V_1 - 0) + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} & -y_{13} \\ -y_{21} & y_{22} & -y_{23} \\ -y_{31} & -y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$I = y_{Bus} V \qquad V = y_{Bus}^{-1} I$$

$$V = Z_{Bus} I$$

y_{Bus} is the admittance matrix:

- It is a square matrix
- The main diagonal contains the self-admittance: the summation of all admittances connected to this node
- The off diagonal contains the mutual admittances between the different nodes
- The main diagonal contains admittances with positive signs, which means a negative value (the admittance itself has a negative value, i.e. –j x)
- The elements in the off diagonal are admittances with negative signs, which means positive values.

Node Elimination general steps

Nodes are classified into main nodes and auxiliary nodes Insignificant nodes are eliminated and the significant ones are maintained

Suppose that we have some main nodes denoted by **"K"** and the auxiliary nodes denoted by **"M“**

Set the associated current components to zero

$$\begin{bmatrix} I_A \\ 0 \end{bmatrix} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_X \end{bmatrix} \qquad I_A = K\,V_A + L\,V_X$$

$$0 = L^T V_A + M\,V_X \qquad V_X = -M^{-1} L^T V_A$$

$$I_A = K\,V_A - L\,M^{-1} L^T V_A \qquad I_A = [K - L\,M^{-1} L^T]V_A$$

$$\underline{I_A = y_{Bus\text{-}new} V_A}$$

$$\boxed{y_{Bus\text{-}new} = K - L\,M^{-1} L^T}$$

مجرد بتجيب $Y_{Bus\ new}$ وتعيد رسم الدائرة

The Reactive Power Control

Balance :

- The balance of reactive power is achieved when the generated reactive power by **synchronous machines and capacitances** is equal to the reactive power of the **loads** plus the **reactive transmission** losses

Imbalance :

- Imbalance in active power causes **frequency fluctuations**-
- Imbalance in reactive power causes a **deviation** of the **voltages** from the desired values

Voltage and Reactive Power

Increasing the **reactive** power **increases** the **voltage**, while **increasing the consumed** reactive power **decrease the voltage**

$$Q \uparrow \Rightarrow V \uparrow$$

The reactive power cannot be transported over long distances in the system, since normally X >> R

Sources of Reactive Power

•Overexcited synchronous machines

•Capacitor banks

•The capacitance of overhead lines and cables

Consumers of reactive power

•Under-excited synchronous machines

•Inductive static loads

•Induction motors

•Shunt reactors

•The inductance of overhead lines and cables

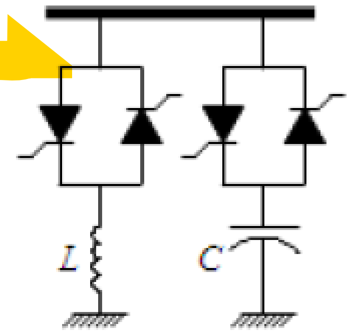
•Transformer inductances

SVC (Static VAR Compensator)

Switching of shunt capacitors and reactors can control the reactive power

If thyristors are used to switch capacitors and/or to control the current through shunt reactors, a fast and step-less control of the reactive power is obtained

Such a device is called SVC (Static VAR Compensator)



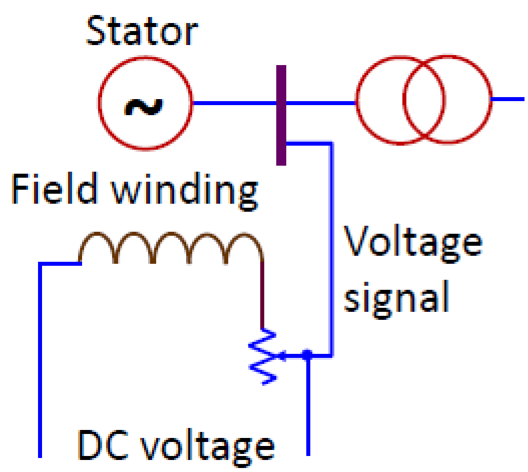
Factors identifying the power system voltage

- Terminal voltages of synchronous machines
- Transmitted reactive power
- Impedances of lines
- Turns ratio of transformers

The generator voltage can be maintained constant using an Automatic Voltage Regulator **(AVR)**

The AVR controls the excitation of the machine so that the voltage is kept constant at the set value

This method cannot easily be used for a single generator when a number of generators run in parallel



Large reactive transmissions cause large voltage drops, therefore, they should be avoided

The production of reactive power should be as close as possible to the reactive loads

The most cost-effective way is to use **shunt capacitors** which are switched according to the load variations

A SVC can be economically motivated if fast response or accuracy in the regulation is required

Shunt reactors must sometimes be installed to limit the voltages to reasonable levels

$\varphi \uparrow$ $V \uparrow$ $P \uparrow$ $\delta \uparrow$

$\varphi \downarrow$ $V \downarrow$ $P \downarrow$ $\delta \downarrow$

ال δ لا يجب ان تزيد عن 90° وتشتغل عند 70 مثلاً أو 60

عشان لو حصلت حاجه .

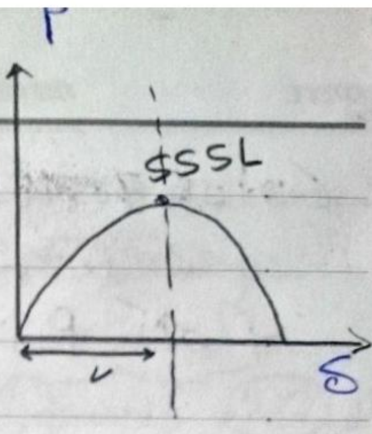
note

$\omega \uparrow$ $P \uparrow$ مع ثبات ال Torque

SSSL → steady state stability limit

يعني حالة الاتزان مستقره

احنا نشتغل في النص الشمال لان النص اليمين unstable



احسن في ال زاويه 90°

Types of Capacitors

-Static Capacitors -Dynamic Capacitors

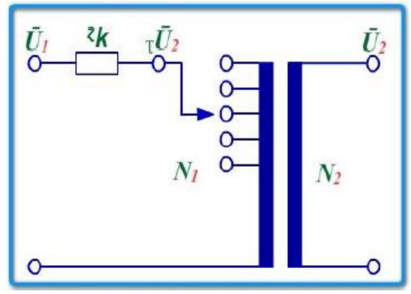
1- Dynamic Capacitors (synchronous generators (compensators))

Are synchronous machines with no load or turbine , just produce or consume **reactive power** by **controlling the excitation** (over : produce // under : consume)

It's **rare** in use, instead capacitors at receiving end are used

Tap changing transformer

by changing the turns ratio of a transformer to control voltage , equipped with number of tabs on the high voltage side , control by switching between these tabs

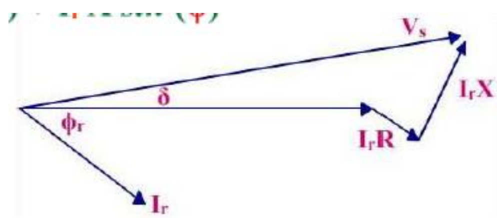


- The switching can be done online during operation
- Normally the taps are placed on the high voltage winding, the upper side
- The lowest current needs to be switched
- The number of turns in the high voltage side is larger, which gives **better regulation** of the variation ratio
- Only **10%** percent of the winding used in tab settings
- Transformers that can't be used online only **change voltage level not voltage variation**

Improving Regulation and PF

$$\Delta V = I_r R \cos\phi + I_r X \sin\phi$$

$$but\ P = V_r I_r \cos\phi \quad ; Q = V_r I_r \sin\phi$$



$$\therefore \Delta V = \frac{PR + QX}{V_r}$$

$$Reg\% = \frac{\Delta V}{V_r}$$

$$\therefore \boldsymbol{Reg\%} = \frac{\boldsymbol{\Delta V}}{\boldsymbol{V_r^2}} = \frac{\boldsymbol{P_r}}{\boldsymbol{V_r^2}}\ [\boldsymbol{R} + \boldsymbol{X}\ \tan\boldsymbol{\phi_r}]$$

Methods to improve the voltage regulation

- Increase the operation voltage $V \uparrow$
- improve the power factor at receiving end $PF \uparrow$
- reduce the resistance of T.L (use double circuit) $R_{TL} \downarrow$
- Reduce the reactance of TL (double circuit-bundle-series capacitor) $X_L \downarrow$

Improve Power Factor

<u>Disadvantages of low power factor</u>

•Higher copper losses $P_{loss_{CU}} \uparrow$

•Poor voltage regulation $\%Reg \downarrow$

•Greater conductor size

•Larger apparent power rating of equipments

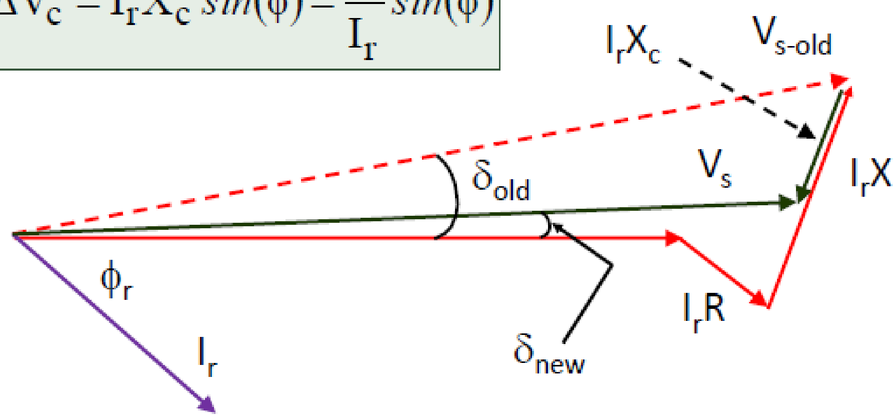
•Reduced handling capacity of the system

To improve the power factor, **parallel** or **series capacitors** are connected

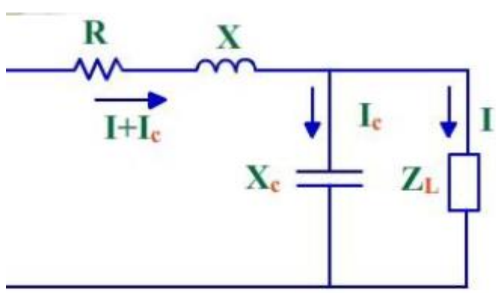
(A) Series Capacitor at receiving end

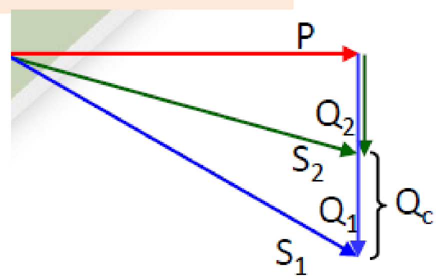
$$\boxed{\Delta V = I_r R\,cos(\phi) + I_r X\,sin(\phi) - I_r X_c\,sin(\phi)}$$

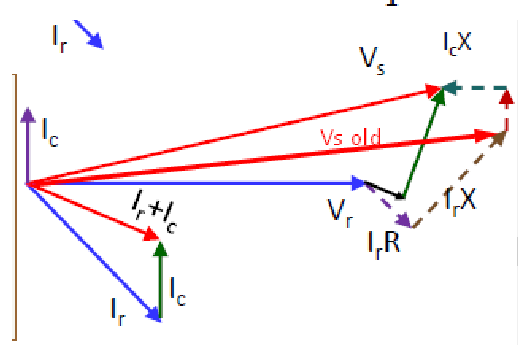
$$\boxed{\Delta V_c = I_r X_c\,sin(\phi) = \frac{Q}{I_r}\,sin(\phi)}$$



(B) Parallel Capacitor (shunt) at receiving end





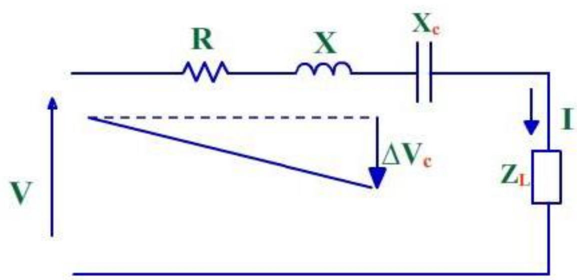
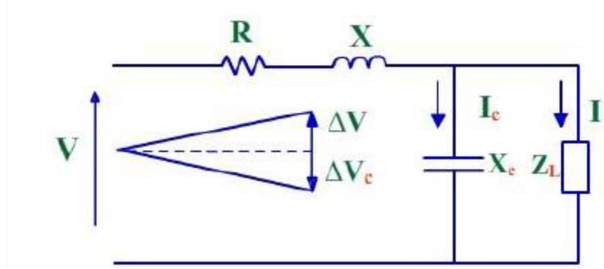


$$\Delta V = I_r R\,cos(\phi) + I_r X\,sin(\phi) - I_c X$$

$$\Delta V_c' = I_c X = \frac{V_r}{X_c} X$$

$$\Delta V_c' = \frac{Q'}{V_r} X$$

Comparison between series and shunt capacitors

Series Capacitors	Shunt Capacitor
In the case of series compensation $\Delta V_{c\text{-}s} = \frac{Q}{I} sin(\phi)$ 	In the case of parallel compensation $\Delta V_{c\text{-}p} = \frac{Q'}{V_r} X$ 
•Sudden increase in the voltage at the capacitor location • Capacitor has to be placed in on the tower •Failure of the capacitor causes a **line break**	•Gradual increase of the voltage •Capacitor is placed on the ground level •Failure of capacitor causes a **voltage reduction**
•The capacitor current Is the load current $\boxed{\Delta V_{c-s} = \frac{Q}{I} sin(\phi)}$	The capacitor current is a part of load current $\boxed{\Delta V_{c-p} = \frac{Q'}{V_r} X}$
Less Expensive	More Expensive
Less Q	Produce 6 times more Q at same voltage

$$\frac{Q'}{V_r} X = \frac{Q}{I} sin(\phi)$$

$$\frac{Q'}{Q} = \frac{V_r}{IX} sin(\phi) \approx \frac{V_r * 0.6}{0.1 V_r} \approx 6$$

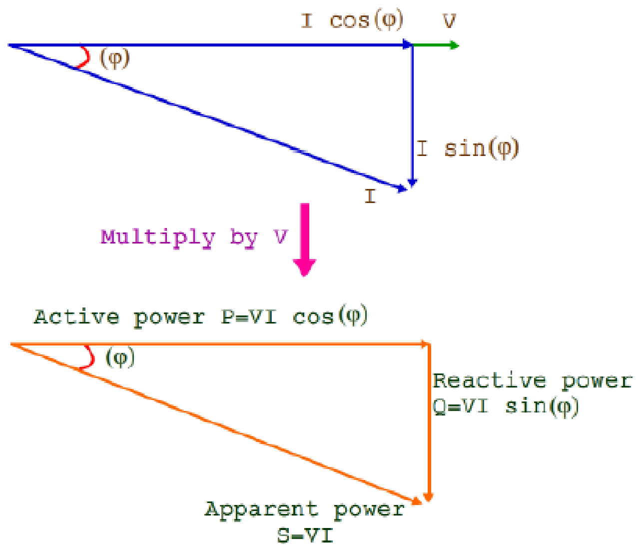
1. To maintain constant load voltage, sending-end voltage to be maintained is low with Series– capacitive arrangement than with Shunt-capacitive arrangement.
2. Shunt-capacitive arrangement reduces the total active power loss while Series–capacitive arrangement does not affect it.
3. Series-capacitive arrangement reduces the total reactive power loss by a large margin as compared to Shunt-capacitive arrangement.
4. Shunt-capacitive arrangement improves the system power factor by a large margin as compared to Series-capacitive arrangement.
5. Load power factor remains always constant with and without any compensation technique.

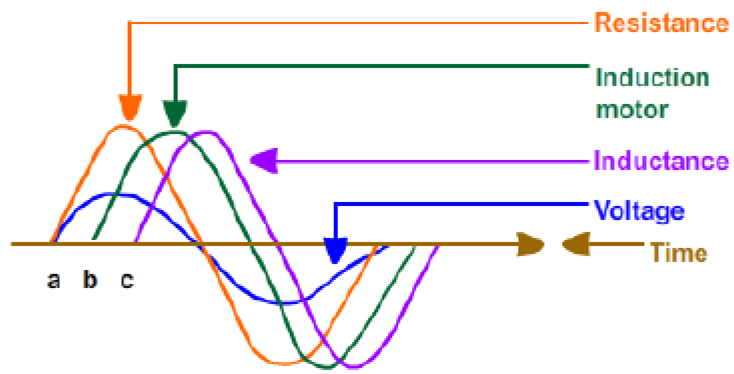
Economics of Power Factor

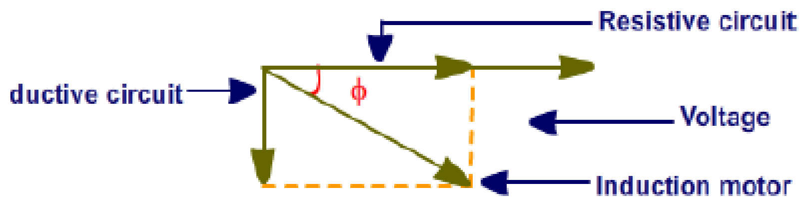
The reactive loads (L and C) shift current causing bad power factor = $\cos\phi = \cos\theta_v - \theta_i$

Poor Power Factor $PF \downarrow$ increase current $I \uparrow$ then $P_{loss} \uparrow$

Phasor diagram and power triangle for lagging power factor

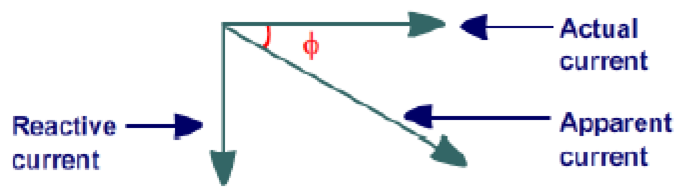






$$PF = \cos\phi = \frac{P(active)}{S(apparent)}$$

$$; \sin\phi = \frac{Q(phantom)}{S(apparent)}$$



$R \rightarrow \phi = 0$

$L \rightarrow \phi = 90$

$C \rightarrow \phi = -90$

$\phi \uparrow \Rightarrow PF \downarrow$

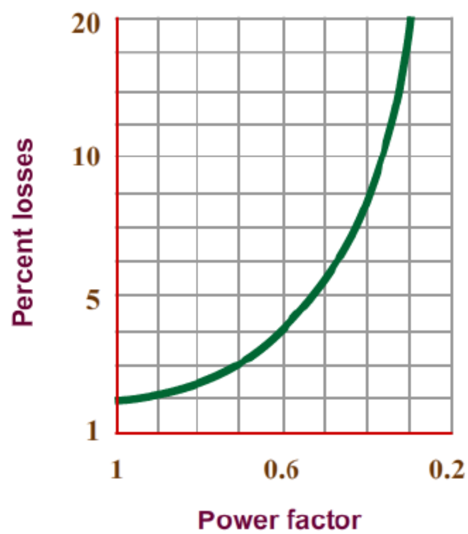
We add reactive power sources to (improve power factor - These effects include improved voltage profiles, enhanced stability, and increased transmission capacity)

Capacitor banks	Static capacitors	Synchronous capacitors
Utility companies and factories place capacitor banks produce a leading pf matches the load's lagging power factor to approach unity it can be switched or **adjusted** according to load **conditions**	The PF correction capacitors are connected in parallel with the utility lines as close as practical to the low-PF loads	can be adjusted to provide varying capacitance to correct for varying PF loads

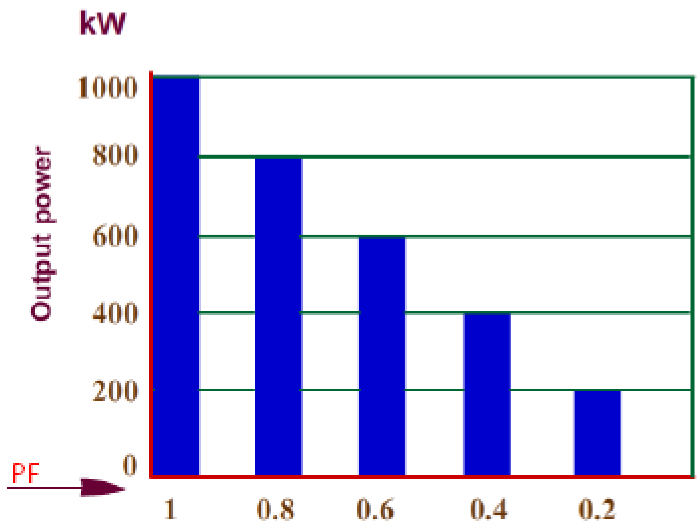
Power Factor Billing

- Charge on monthly kvar hours
- An increasing penalty for decreasing PF
- A straight charge for the maximum value of KVA used during the month
- A penalty for PF below a predetermined value or a credit for PF above a predetermined value

Relation between **PF** and **Percentage losses**



PF and **Transformer** output



$$VR = \frac{k\,var_{cb} \cdot Z_t}{kVA_t}$$

VR: per cent voltage rise

$Kvar_{cb}$: sum of capacitor kvar ratings

Z_t: percent reactance of the supply transformer(s)

kVA_t: kVA rating of the supply transformer(s)

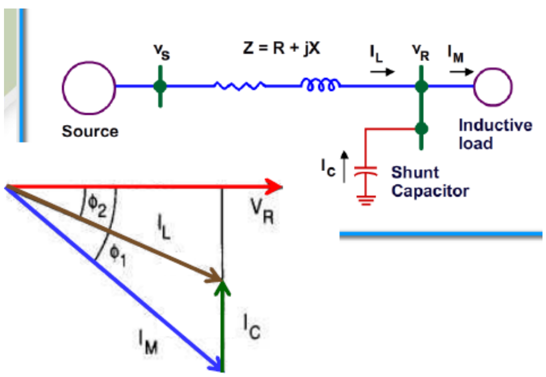
On-Site Power Factor Correction

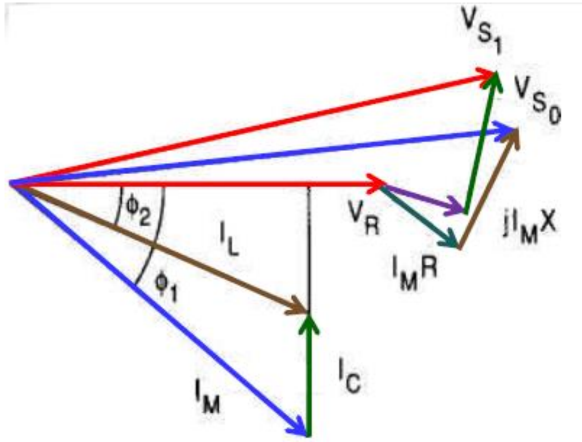
Improve power factor in 2 ways

1- Reduce the reactive energy by **eliminating** low PF loads, e.g. **unloaded motors** and transformers
2- **Apply** external **compensation capacitors** or other devices to correct the low-PF condition

Compensating capacitors:

PF correction capacitors perform the function of an **energy-storage** device Instead of transferring reactive energy back and forth between the **load** and the **power source**, the **magnetizing current reactive energy** is **stored** in a **capacitor** at the load





LIMIT

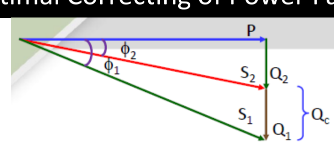
Correction to **0.85** will **satisfy** many requirements No economic advantage is likely to result from correcting to 0.95 or greater

NOTES

- **Overcorrecting** a load by placing too many PF correction capacitors **can reduce the power factor** after reaching unity, and cause **uncontrollable overvoltages** in **low-kVA-capacity power sources**
  - If the capacitors are **switched on** and **off**, they will create significant **impulses** of their own

Solution : Such resistors are connected momentarily in series with the capacitors
After a brief delay (0.5 s or less), the resistors are short-circuited, connecting the capacitors directly across the line

Optimal Correcting of Power Factor



$$\text{Annual saving} = A(S_1 - S_2) = A\left[\frac{P}{\cos(\varphi_1)} - \frac{P}{\cos(\varphi_2)}\right] \quad \text{(L.E./year)}$$

"A" is the cost of kVA (L.E./ kVA), "S_1" and "S_2" are the apparent power before and after correction

The price of the capacitor bank depends on the reactive power capacity

So ,

$$\textit{Net Annual saving} = Sav_{net} = A\left[\frac{P}{\cos(\varphi_1)} - \frac{P}{\cos(\varphi_2)}\right] - \frac{B.b}{100}(Q_1 - Q_2)$$

$$Sav_{net} = A.P\left[\frac{1}{\cos(\varphi_1)} - \frac{1}{\cos(\varphi_2)}\right] - CP(\tan(\varphi_1) - \tan(\varphi_2))$$

B: *the unit cost of the capacitor reactive power cost*

$$\frac{b}{100} \textit{ the annual amount}$$

$$C: C = B.\frac{b}{100}$$

To get the **optimal** value

$$\frac{\partial Sav_{net}}{\partial \phi_2} = 0:$$

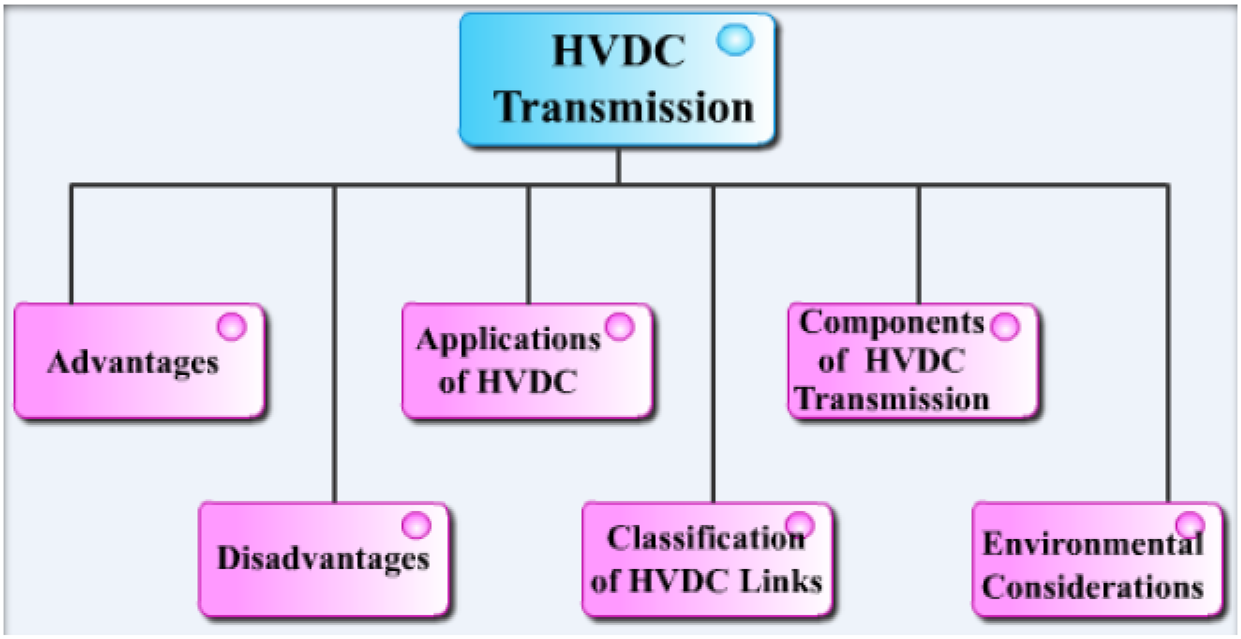
$$0 = A.P\left[-\frac{\sin(\varphi_2)}{\cos(\varphi_2))^2}\right] - CP\left(-\sec^2(\varphi_2)\right)$$

$$0 = A\left[-\frac{\sin(\varphi_2)}{\cos(\varphi_2))^2}\right] + C\left[\frac{1}{\cos(\varphi_2))^2}\right]$$

Or $A\sin(\varphi_2) = C$

Thus $\sin(\varphi_2) = \dfrac{C}{A}$

And $\cos(\varphi_2) = \sqrt{1 - \left[\dfrac{C}{A}\right]^2}$



Advantages of HVDC Transmission

1.Only two conductors per circuit are needed, consequently, dc transmission towers carry less conductor dead weight, and they can be smaller and less costly to fabricate

2. No skin effect: lower resistance of a conductor

3. A dc transmission link has no stability problem

4. No Ferranti effect (no limitations for the lines.

5. Using the HVDC transmission, the prime mover speed need not be confined to correspond to 50 or 60 Hz, but could rather be chosen for best economy.

6. The dc itself does not require reactive power.

7. For the same amount of transmitted power over the same size conductors, the dc line losses are smaller than with ac transmission lines.

Disadvantages

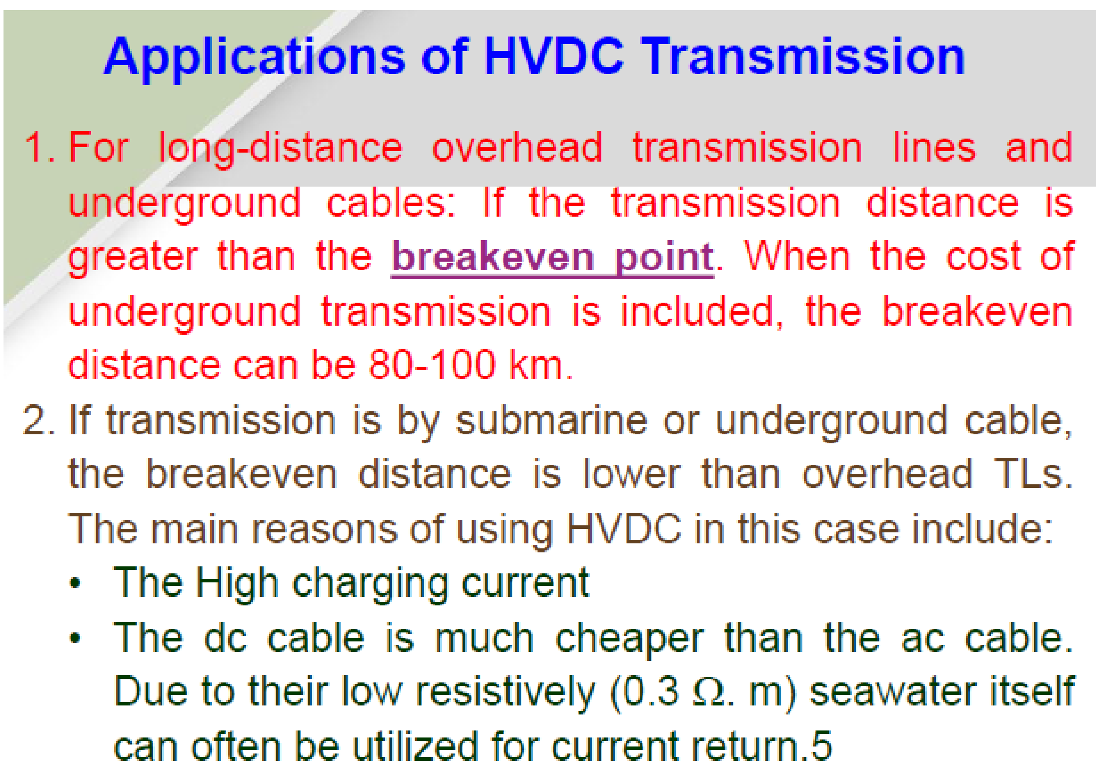
1.The lack of HVDC circuit breakers is regarded as limitation of HVDC transmission. In ac circuits, circuit breaker takes advantage of the current zeros occurring twice per cycle.

2.The reliability and maintenance of converters are major problems for dc systems

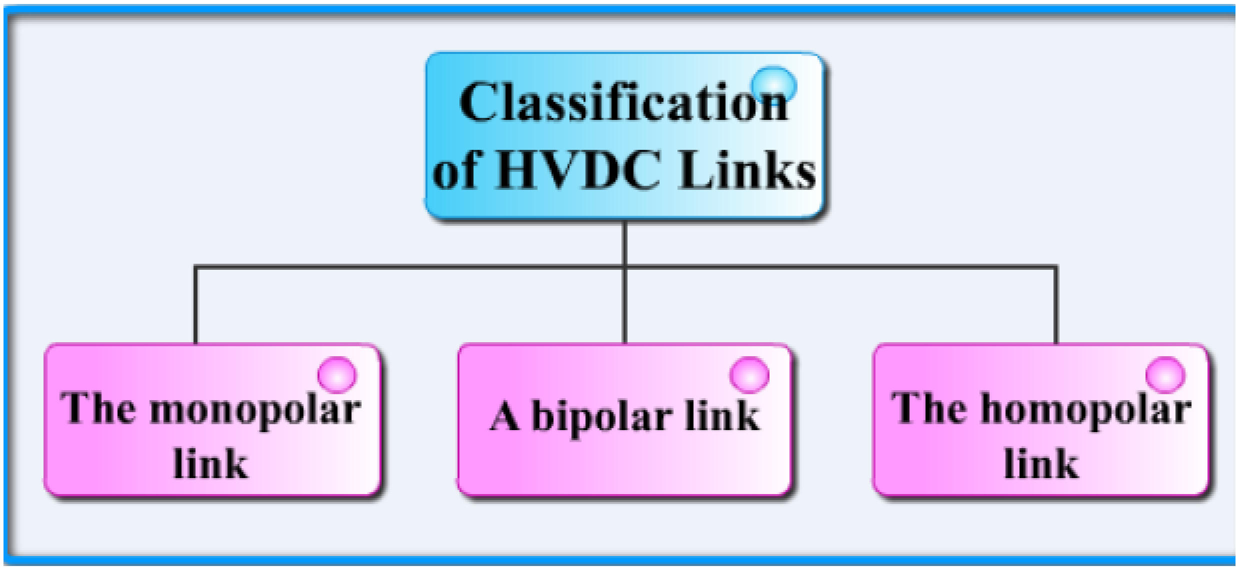
3.Production of harmonics due to converter operation and need for filters in both sides of dc systems

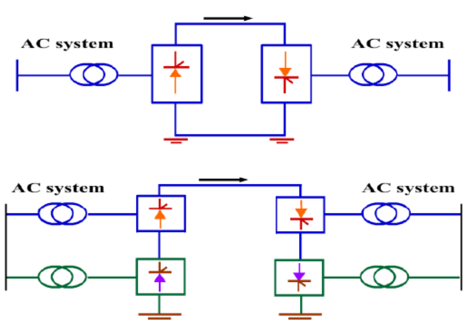
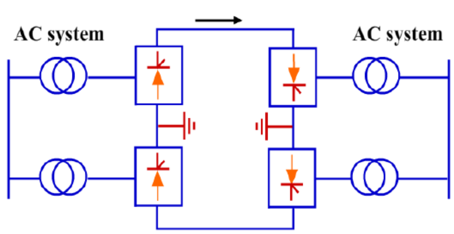
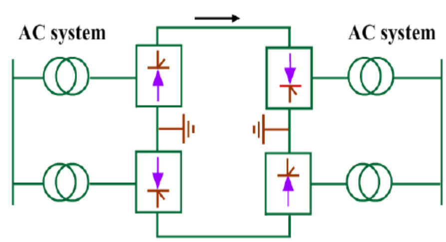
4.High cost of conversion equipment.

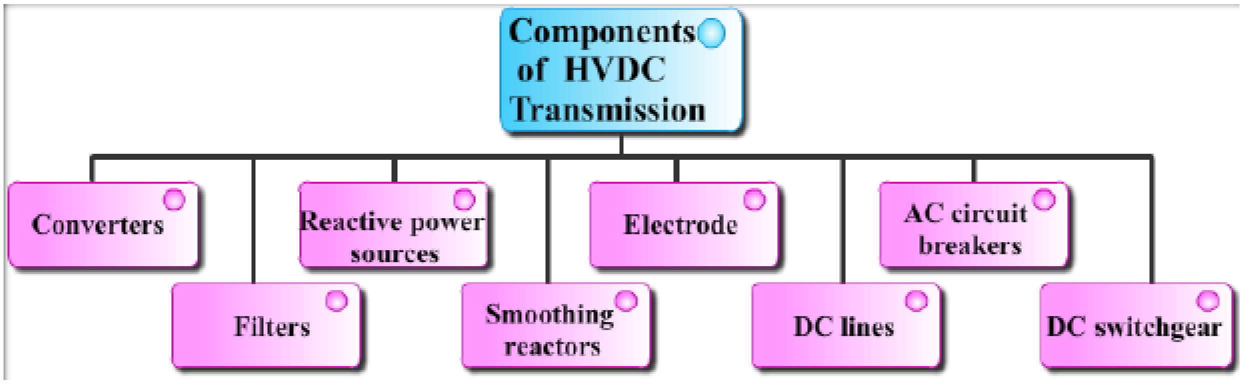
5.Complexity of control



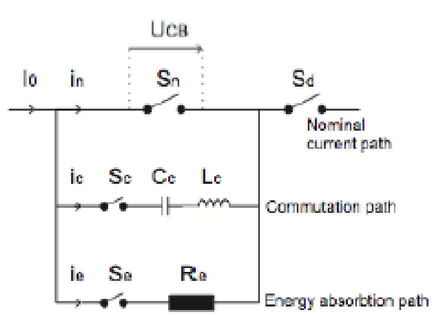
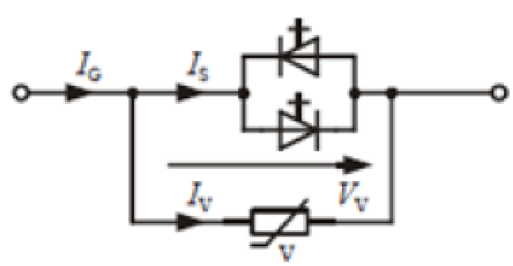
3. Some a.c. power systems can not synchronized to neighbouring networks even with their small distances (different frequencies)
4. Deliver energy from remote energy sources, where generation has been developed at remote sites of available energy
5. Increasing the power transfer capability of existing a.c. transmission by conversion to d.c. transmission.
6. Power flow control through voltage rather than angles
7. Two ac systems, which have different strategies of control technique may tied together through a dc interconnection.



Monopolar	Bipolar	Homopolar
•The monopolar link has **one conductor** (usually of negative polarity) and uses ground and sea return •This type of configuration may also be the first stage in the development of bipolar systems •Instead of ground return, a metallic return may be used in situation, where the earth resistivity is high  	•A bipolar link consists **of two conductors**, one positive and the other negative. •The neutral point given by the junctions between the converters is grounded at one or both ends •Each terminal has two converters with equal rated voltages (series on dc side and parallel on ac side) •It is considered to be equivalent to a double-circuit ac transmission line •It causes considerably **less harmonic interference** on nearby facilitate than the monopolar system •**Reversal of power-flow** direction is achieved by changing polarities of the two poles through controls.  	•The homopolar link has two or more conductors having the **same polarity (usually negative)**, and always operates with ground return •The main advantageous of homopolar is that a negative polarity causes **less** radio interference due to **corona** 

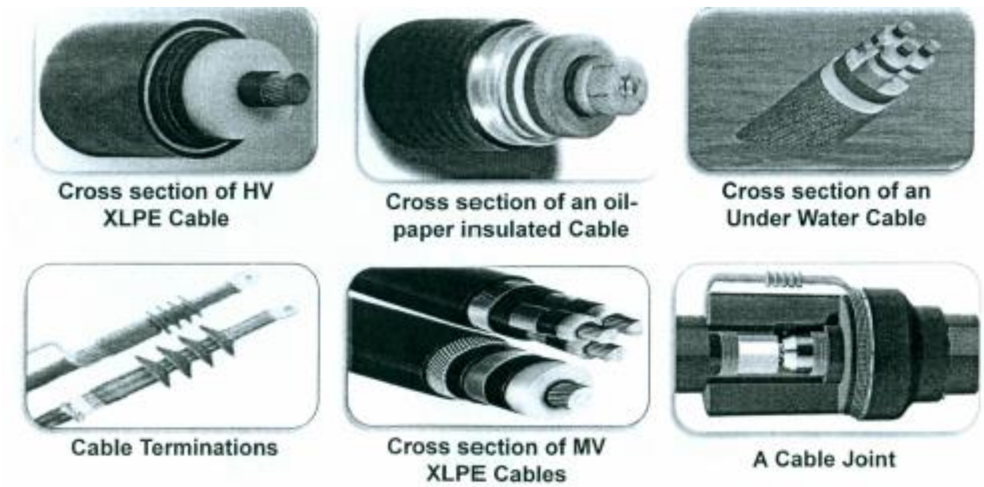


CONVERTERS	Conversions from ac to dc (Rectifier station) and from dc to ac (Inverter station) are performed by converters, and consists of valve groups and transformers with tap changers. A point-to-point transmission requires two converters. The role of rectifier and inverter stations can be reversed (resulting in power reversals) by suitable converter control.
FILTERS	There are three types of filters used: 1. Ac filters : both low and high frequency filters are used to suppress the ac current harmonics. 2. Dc filters : both low and high frequency filters are used for the filterin g of dc harmonics. 3. High frequency filters, these are connected between the converter transformer and the station ac bus to suppress any high frequency currents.
REACTIVE POWER SOURCES	Converter stations require reactive power supply that is dependent on the active power loading (about 50 to 60% of the active power). Fortunately, the ac filters provide part of this reactive power requirement. In addition, synchronous condensers and static var systems are used depending on the speed of control desired.
SMOOTHING REACTORS	A sufficient large series reactor is used on dc side to smooth dc current and also for protection . The reactor is designed as a linear reactor and is connected on the line side, neutral side, or in both in the line and neutral side.
ELECTRODE	Most dc links designed to use earth as neutral conductor for at least short periods of time. The connection to the earth requires a large-surface-area conductor to minimize current densities and surface voltage gradients. This conductor is referred to as an electrode.
DC LINKS	They may be overhead lines or cables. Except for the number of conductors and spacing required, dc lines are very similar to ac lines.
AC CIRCUIT BREAKERS	For clearing faults in the converter transformer and for taking the dc link out of service, circuit-breakers are used on the ac side. They are not used for clearing dc fault, since this fault can be cleared more rapidly by the converter control.

DC SWITCHGEAR	**This is usually a modified ac equipment used to interrupt small dc currents. Dc breakers are also used, for interruption of rated load currents.**	
DC Circuit Breaker	Electromechanical circuit breaker	Solid state circuit breaker
	(1) inverse voltage generating method, (2) divergent current oscillating method, **(3) inverse current injecting** method: n this type of breaker, current zero can be created by superimposing an inverse current (of high frequency) on the input current by **dis-charging a capacitor** (that was pre-charged) **through an inductor**.) The cost of components required for an electromechanical DC circuit breaker would not be significantly higher than that of an AC circuit breaker. Electromechanical HVDC circuit breakers are available up to **500 kV, 5 kA** and have a fault-clearing time of the order of **100 ms.** • The **nominal current path** is where DC current passes through and the switch is closed during normal operation The **commutation path** consists of a switch and a resonant circuit with an inductor and a capacitor and is used to create the inverse current The **energy absorption path** consists of a switch and a varistor 	- can interrupt current much faster - based on Integrated Gate Commutated Thyristors (**IGCT**), which compared to IGBT (bipolar thyristors) **have lower on-state losses** - Current flows through the IGCT and in order to interrupt, the IGCT is turned off. Once that happens, voltage quickly increases until a varistor (zinc Oxide) (that is in parallel to the thyristor) starts to conduct. The **varistor is designed to block voltages above the voltage level** of the system. The main **disadvantages** of these types of circuit breakers are the **high on-state losses** and the **capital costs.** - In this topology, parallel connections are unnecessary - **Advantage :** the power losses per circuit breaker are very low in the solid-state case. 

Underground cables:

Cable lengths , joints and terminations :



Classification :

Material & Design :

- Oil filled (impregnated) cables

-Extrude solid cables (PE : Polyethylene / Ethylene propylene rubber (EPR))

- Gas insulated cables

Voltage level :

- Low Voltage $U_n = 1kV$
- Medium Voltage $U_n > 4\ kV$
- High Voltage $U_n\ 45{:}\ 150\ kV$
- Extra high voltage $U_n > 150\ kV$

Cable Components

Conductors -- copper or aluminum

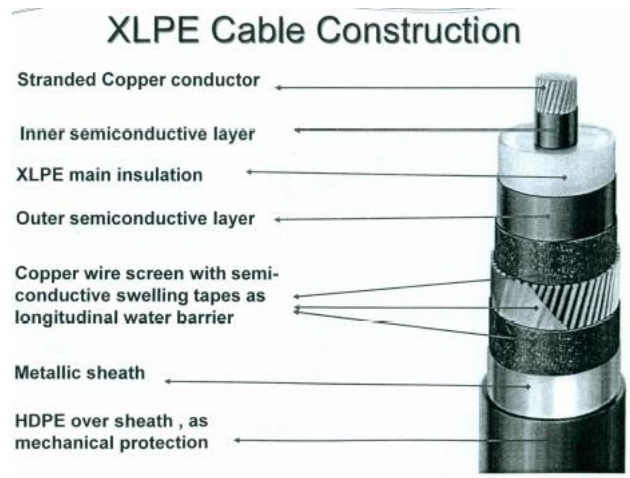
Insulation – oil paper or polymeric

Field limiting layers - semiconducting materials

Metallic covering -- lead or aluminum or copper

Outer coverings and corrosion protection – PVC or LDPE

Water tightness of cable (swelling tape)

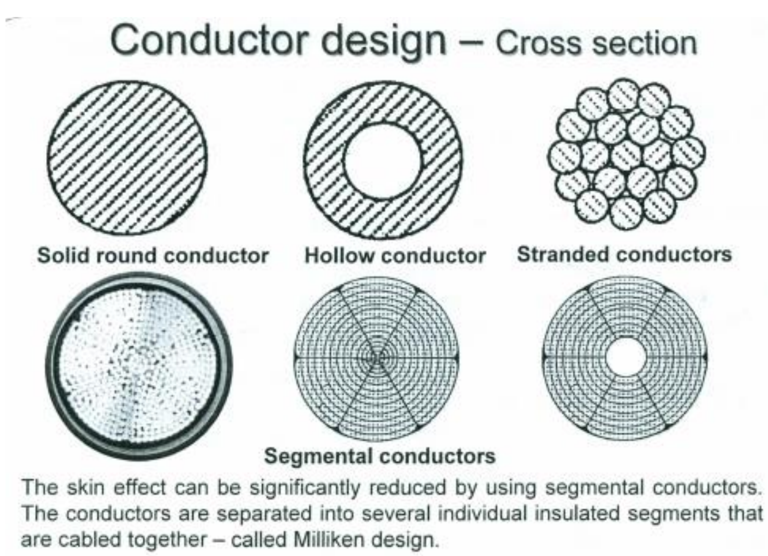


Note : difference in electrical conductivity – between conductors and insulation – nearly **20 orders**

Conductor

Copper or aluminum:
Depending on (current power rating, single solid or hollow, or multiple stranded conductors are used.)

- Conductivity of aluminum is **62%** for same resistance cross-section of aluminum =**1.6** of copper
- Copper has **3 times tensile** strength of aluminum
- Weight of Meter of aluminum = 48% of meter copper (aluminum density = 30% of copper)
- Cost of overlying material must be considered
- Aluminum creeps easily (deform or relax over time underload)



Insulation Designs

(PE : Polyethylene / Ethylene propylene rubber (EPR)) [[Paper /Natural rubbers (NR) /Silicone rubber (SR) / butyl rubber (BR) /Ethyl vinyl acetate (EVA)]]

Most of modern PE is XLPE

- Almost all polymeric
- insulation compounds have proprietary additives to improve their expected life
- (TR-XLPE) tree retardant : to slow the growth of water trees
- **paper-oil** insulation is not removed from the list; it is still a preferred choice at **EHV**
- significant number of these cables are
- .performing well in the existing power grids in specific applications.
- Electrical isolation is required from start (source side) to the end (load side) with solid insulation , unlike in over head lines.

Polyethylene cables

.;- Low density polyethylene

.:. High density' polyethylene

.:. Cross-linked polyethylene (XLPE)

.:. Tree-Retardant cross-linked polyethylene (TR-XLPE)

<u>Common to all types</u>

.:. Low Permittivity

.:. Low Tanδ: Low Losses

.:. Very High dielectric strength (prior to aging)

.:. Easy to process/extrude

Operating Temperatures

Permissible conductor temperatures for high and extra high voltage cables

Dielectric Used	Impregnated paper	LDPE	HDPE	XLPE	EPR
Operating temperature in °C	85/90	70	80	90	90
Short-circuit temperature in °C	160/180	150	180	250	250

Although the operating temperature depends on the current carrying capacity; the type of insulation used limits the allowable operating temperature.

Shielding , sheaths, jackets and armors

FIELD LIMITING LAYERS

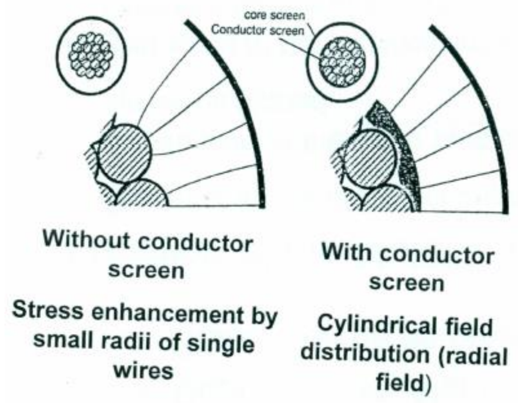
To ensure a defined cylindrical field and to withstand the field strengths that occur} all cables for > 6 kV, independent of their type of designs~ require field limiting or field smoothing layers.

(Inner semi conductive layer (conductor shield) -- external semi conductive layer (core shield))

- It's required in (stranded , medium & high voltage cables)
- it provide smooth radial electric field within insulation
- It's resistivity must be below 100 Ω/m but modern has reached below 10 Ω/m

BENEFITS

1- Equalization and reduction of the electrical stress in the cable dielectric by preventing local field enhancement in non-homogeneous areas such as the individual wires of the conductor or screen.
2- Eliminate the effect of the individual wires on the field distribution,
3- Prevention of the formation of gaps or voids between the current carrying components of the cable (conductor, screen and metal sheath) and the insulation layer due to mechanical stress



Metallic Covering

) (copper , lead , aluminum / steel-sheath as pressure pipes)

Fuction of metallic cover

- Return of the capacitive charging current under operating conditions.
- Conduction of the earth fault current in the case of a fault until the system is tripped.
- Reduction of the electrical influence on the cable surroundings in the case of a fault, e.g. an earth fault.
- Provision of protection against accidental contacts.
- Mechanical protection of insulation while allowing the cable to. bend sufficiently.

Outer covering and corrosion protection

- (HDPE) provides good **mechanical protection** an excellent resistance to **abrasion**
- With low **moisture penetration**

  that's why HDPE has displaced the common PVC (polyvinyl chloride)
-

Underground Layout and Construction

Direct Buried Cables

Underground distribution power cables can be installed directly in a trench (direct burial) or in a duct In some instances, power cables can be installed with telephone, gas, water, or other facilities Direct burial of power cables is commonly used in low-density residential areas The main advantage of direct burial installation is its **low cost**

Cable in Ducts

Ducts or conduits are normally used under roadways, or in locations where mechanical or other types of damage may be expected. Conduit installation is expensive and complex and the conduit type should be selected carefully

Manholes

Manholes are typically built at the splices as a way for workers to install **cables** or other **equipment**, provide **test** points, and perform routine or **maintenance** The dimensions of the manholes should accommodate the conduits and cables entering the manhole and any equipment installed, such as transformers or protective equipment

Partial Discharge Test

Gas-filled voids are found in the insulation and at the juncture of dielectric and conductive sheaths of cables The breakdown of this gas produces a phenomenon known as partial discharge Partial discharge occurs at these voids because the breakdown strength of the gas within voids is much less than that of the typical cable insulation material In fact, discharges may occur at voltages lower than the operating voltage of the cable The level of voltage at which partial discharge occurs first is called the **discharge inception voltage**

Tanδ

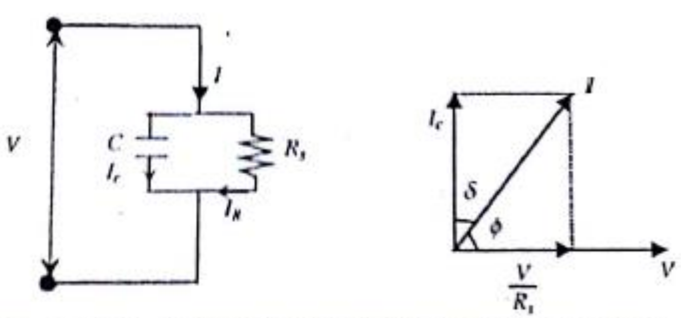


Figure 14.17 Equivalent circuit of cable and its phasor diagram.

$$\delta \approx \tan\delta \approx \sin\delta \approx \sin(90-\phi) = \cos\phi$$

ram,

$$\frac{V/R_s}{V\omega C} = \tan\delta \quad \text{or} \quad \frac{V}{R_s} = V\omega C \tan\delta$$

ric power loss

$$P_l = V^2\omega C \tan\delta = V^2\omega C\delta \text{ watt}$$

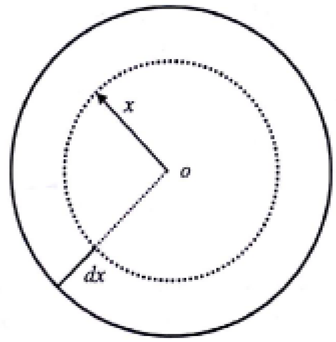
high voltage test

high voltage test (water immursion test or the all cable test) fel water immursion test bna5od 3yna mn el cable tolha 3 m we n8mr el cable fel mya 24 h w b3d keda nslt ghd mabeen el conductor wel mya el all cable test btslt aksa ghd yt7mlo el cable lmdt 5 min w tshof hy7sl break down lel 3azl wla la

el wrapping test

el wrapping test bt3m wrapping lel conductor 7walen m7wro (5-6) turns we b3d t3mlo unwrap w tkrar el process 3 mrat we da by5br ductility bta3 el conductor

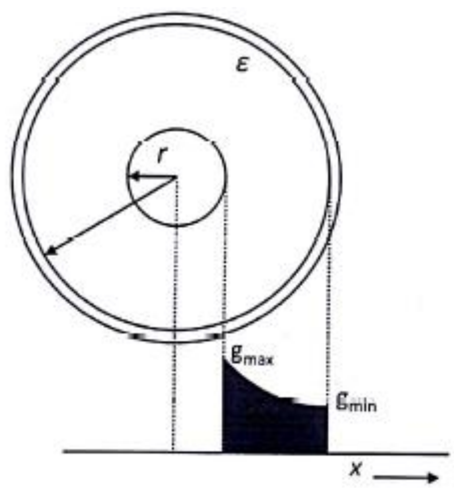
Insulation Resistance



$$dR_s = \frac{\rho dx}{2\pi x l} \qquad R_s = \frac{\rho}{2\pi l}\int_r^R \frac{dx}{x} = \boxed{\frac{\rho}{2\pi l}\ln\frac{R}{r}}\ ohm$$

Per unit length $R_s = \frac{\rho}{2\pi}\ln\frac{R}{r}\ ohm/m$

Electrostatic stress in a single core cable



$$E_x = \frac{q}{2\pi\varepsilon x}\ volt/m$$

$$V = -\int_R^r E_x \, dx = \boxed{\frac{q}{2\pi\varepsilon} \ln\frac{R}{r}}$$

$$\therefore \quad \boxed{E_x = \frac{V}{x \ln(R/r)}}$$

Max (x= r)	Min (x = R)
$E_{max} = \frac{V}{r \ln(R/r)}$	$E_{min} = \frac{V}{R \ln(R/r)}$

To get optimum value of radius

$$\frac{dE_{max}}{dr} = 0$$

$$r * \frac{r}{R} * \left(-\frac{R}{r^2}\right) + \ln\frac{R}{r} = 0$$

$$\ln\frac{R}{r} = 1 \quad or \quad \frac{R}{r} = e$$

$$thus\ R = 2.718\, r$$

$$\boxed{E_{max} = \frac{V}{r} = \frac{V_e}{R}}$$

$$the\ maximum\ stress\ allowed = \frac{dielectric\ strength}{safety\ factor}$$

Grading of cables

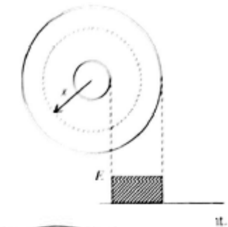
To uniformly distribute electric stress and utilize the insulator (higher voltage for same size)

Classified to

(a) Capacitance grading (more than one dielectric material)

1- **For same safety factor**
2- **For same maximum stress**

(b) Intersheath grading (the same dielectric material potentials are held at certain radius by metal sheath)



(a) <u>Capacitance grading (more than one dielectric material)</u>

Ideally we want to have uniform gradient like in photo

So that $E_x = \frac{q}{\epsilon 2\pi x} = k$

We can't do it practically but to improve it we use assume 3 materials as shown

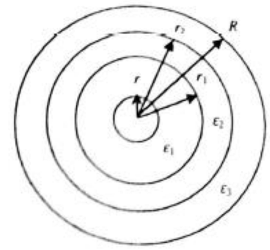


Figure 14.8 Capacitance grading.

1- **For same safety factor**

$$\frac{q}{\epsilon_1 2\pi r} = \frac{G_1}{F} \quad \& \quad \frac{q}{\epsilon_2 2\pi r_1} = \frac{G_2}{F} \quad \& \quad \frac{q}{\epsilon_3 2\pi r_2} = \frac{G_3}{F}$$

$$q = \epsilon_1 2\pi r \frac{G_1}{F} = \epsilon_2 2\pi r_1 \frac{G_2}{F} = \epsilon_3 2\pi r_2 \frac{G_3}{F}$$

$$\therefore \boldsymbol{\epsilon_1 r\, G_1 = \epsilon_2\, r_1 G_2 = \epsilon_3 r_2 G_3}$$

$$but\; r < r_1 < r_2 \quad \therefore \quad \boldsymbol{\epsilon_1 G_1 > \epsilon_2\, G_2 > \epsilon_3 G_3}$$

$$V = \int_r^{r_1} E_1\, dx + \int_{r_1}^{r_2} E_2 dx + \int_{r_2}^{R} E_3\, dx$$

$$= \boldsymbol{\frac{q}{2\pi}\left(\frac{1}{\epsilon_1}\ln\left(\frac{r_1}{r}\right) + \frac{1}{\epsilon_2}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{\epsilon_3}\ln\left(\frac{R}{r_2}\right)\right)}$$

remember you can replace q with

$$q = \epsilon_1 2\pi r \frac{G_1}{F} = \epsilon_2 2\pi r_1 \frac{G_2}{F} = \epsilon_3 2\pi r_2 \frac{G_3}{F}$$

2- **For same maximum stress**

$$E_{max} = \frac{q}{\epsilon_1 2\pi r} = \frac{q}{\epsilon_2 2\pi r_1} = \frac{q}{\epsilon_3 2\pi r_2}$$

$$so\,,\ \ \epsilon_1 r = \epsilon_2 r_1 = \epsilon_3 r_2$$

$$but\ \ r < r_2 < r_2\ \ , \boldsymbol{\epsilon_1 > \epsilon_2 > \epsilon_3}$$

$$\therefore \boldsymbol{V = E_{max}\left(\, r\, ln\left(\frac{r_1}{r}\right) + r_1\, ln\left(\frac{r_2}{r_1}\right) + r_2\, ln\left(\frac{R}{r_2}\right)\right.}$$

thus for the same maximum stress the material having the highest permittivity should be placed near to the conductor and so on.

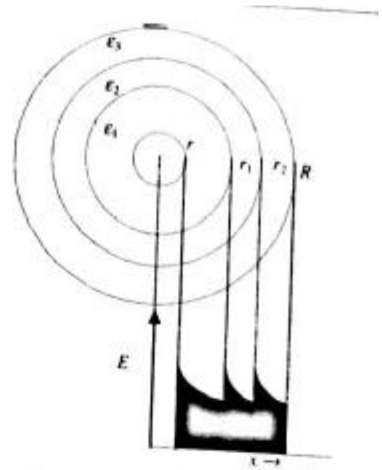


Figure 14.9 Stress distribution.

(b) Intersheath grading (the same dielectric material potentials are held at certain radius by metal sheath)

$$E_{max}1 = E_{max}2 = E_{max}3$$

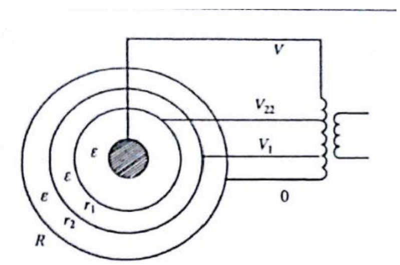


Figure 14.10 Intersheath grading.

$$\frac{V - V_2}{r ln\left(\frac{r_1}{r}\right)} = \frac{V_2 - V_1}{r_1 \ln\left(\frac{r_2}{r_1}\right)} = \frac{V_1}{r_2 \ln\left(\frac{R}{r_2}\right)}$$

$$and\ \frac{r_1}{r} = \frac{r_2}{r_1} = \frac{R}{r_2} = \alpha$$

$$\therefore\ \frac{V - V_2}{r} = \frac{V_2 - V_1}{r_1} = \frac{V1}{r_2}$$

We get

$$V_2 = V_! \frac{r_1 + r_2}{r_2} = \ V_1 \frac{1 + \left(\frac{r_2}{r_1}\right)}{\frac{r_2}{r_1}} = V_1 \left(\frac{1+\alpha}{\alpha}\right)$$

$$\therefore V = V_2 + \frac{rV_1}{r_2} = V_1 \frac{(\,1 + \alpha + \alpha^2)}{\alpha^2}$$

$$\boldsymbol{E_{max}} = \frac{V_1}{r_2 \ln\left(\frac{R}{r_2}\right)} = \frac{V\alpha^3}{(1 + \alpha + \alpha^2) R \ln \alpha}$$

$$E'_{max} = \frac{V}{r ln\left(\frac{R}{r}\right)}$$

$$\therefore \frac{\boldsymbol{E_{max}}}{\boldsymbol{E'_{max}}} = \frac{\mathbf{3}}{\mathbf{1}+\boldsymbol{\alpha}+\boldsymbol{\alpha^2}}$$

$$\boldsymbol{V} = E_{Max}\, ln\alpha\, (r + r_1 + r_2) = \boldsymbol{E_{Max}}\ \boldsymbol{ln\alpha\, r}\,(\mathbf{1}+\boldsymbol{\alpha}+\boldsymbol{\alpha^2})$$

Disadvantage

(a) Non availability of varying permittivity of insulating materials.

(b) Change in the permittivity with time, which changes the distribution of stress that may lead to rapture of insulating material at normal working voltage.

(c) Damage of intersheath during laying or due to aging may lead to severe stress.

(d) Charging current flows through the intersheath which may damage the cable due to overheating.

(e) There may be resonance problem in intersheath grading due to inductance of transformer and capacitance of cable.

(f) Grading may not be economical in low-voltage cables.

Due to these reasons, grading is avoided in modern practice in favour of oil- and gas-filled cables.

CAPACITANCE

Capacitance of single-core cable

$$E_x = \frac{q}{\epsilon\, 2\pi x}$$

$$V = -\int_R^r E\, dx = \frac{q}{\epsilon\, 2\pi} \ln\left(\frac{R}{r}\right)\ volt$$

$$C = \frac{q}{V} = \frac{(2\pi\epsilon)}{\ln(\frac{R}{r})}$$

This capacitance is much more than the overhead transmission line due to the following reasons:

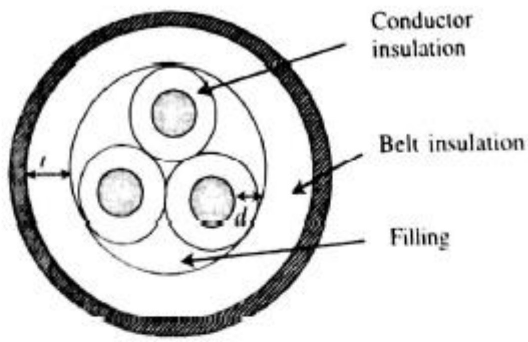
(a) High value of permittivity of insulating material

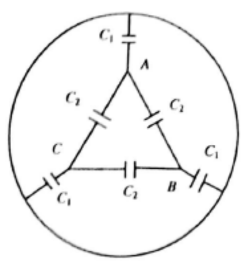
(b) Distance between the core and the earthed sheath is small

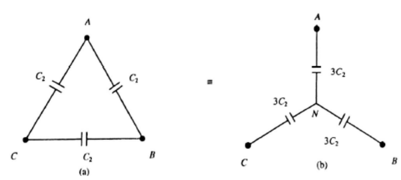
(c) Small distances between the cores (phases) itself.

<u>Capacitance of three-core cable</u>

Belt insulation is required cuz for operating voltage V , conductor insulation is only suitable for V/2 whereas the voltage between conductor and sheath is $\frac{V}{\sqrt{3}}$







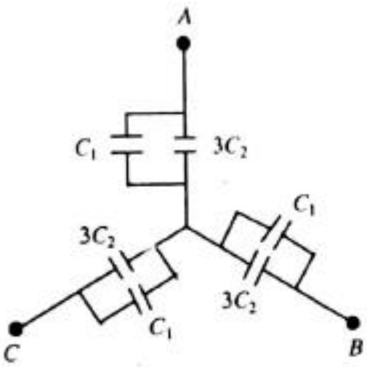


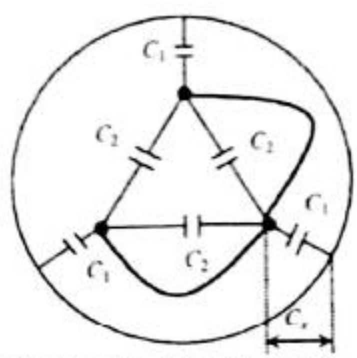
Figure 14.14 Equivalent phase-to-sheath capacitance.

$$C_0 = C_1 + 3C_2$$

$$C_0 = \frac{0.0298\varepsilon_r}{\log_{10}\left[1 + \frac{T+t}{d}\left(3.84 - 1.70\frac{t}{T} + 0.52\frac{t^2}{T^2}\right)\right]} \ \mu\text{F/km}$$

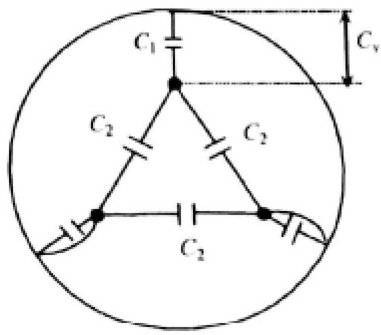
Methods of calculating capacitances

1. Measure the capacitance C_x between the sheath and all the three conductors joined together, as shown in Figure 14.15(a). With this arrangement, the entire conductor-to-sheath capacitances will be in parallel, therefore $\boxed{C_x = 3C_1}$.



(a) Measuring the capacitance C_x

2. Connect any two cores to the sheath and measure capacitance C_y between remaining conductor and sheath as, shown in Figure 14.15(b). With this arrangement, $\boxed{C_y = C_1 + 2C_2}$.



(b) Connecting two cores to the sheath

From method 1 and 2

$$\boldsymbol{C_2} = \frac{C_y - \left(\frac{C_x}{3}\right)}{2} = \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{C_y} - \frac{\boldsymbol{C_x}}{\mathbf{3}}\right)$$

$$therefore\,,\ \boldsymbol{C_0} = C_1 + 3C_2 = \frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{C_y} - \frac{\mathbf{1}}{\mathbf{6}}\ \boldsymbol{C_x}$$

3. Measure the capacitance C_z between two conductors by means of a Schering bridge and connecting the third conductor to the sheath to eliminate one of the C_1s, as shown in Figure 14.16. Thus,

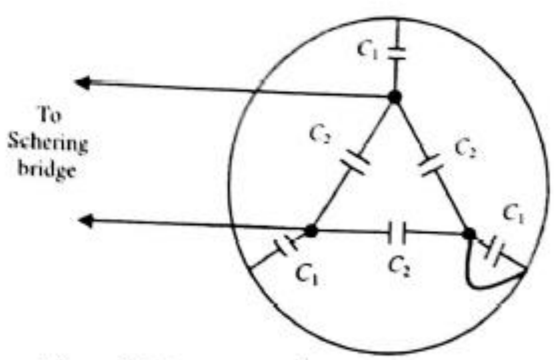


Figure 14.16 Measurement by Schering bridge.

$$C_z = C_2 + \frac{1}{2}(C_1 + C_2) = \frac{1}{2}\ (C_1 + 3\ C_2)$$

Or

$$C_0 = 2\ C_z$$

Equivalent Π and T of long TL

Nominal T representation

$$\boxed{Y' = Y\frac{\sinh(\theta)}{\theta}}$$

$$A = D = 1 + \frac{Z'Y'}{2} = \cosh(\theta)$$

$$\frac{Z'Y'}{2} = \cosh(\theta) - 1$$

$$\frac{Z'}{2}\left(Y\frac{\sinh(\theta)}{\theta}\right) = \cosh(\theta) - 1$$

$$\frac{Z'}{2} = \left(\frac{\cosh(\theta) - 1}{\sinh(\theta)}\right)\frac{\theta}{Y}$$

$$\frac{Z'}{2} = \frac{\theta}{Y}\left(\frac{\tanh(\frac{\theta}{2})}{\frac{\theta}{2}}\right)\frac{\theta}{2} = \frac{\theta^2}{2Y}\left(\frac{\tanh(\frac{\theta}{2})}{\frac{\theta}{2}}\right)$$

$$\boxed{Z' = Z\left(\frac{\tanh(\frac{\theta}{2})}{\frac{\theta}{2}}\right)}$$

Faculty of Engineering – Tanta University

Electrical Power and Machines Engineering Department

DERIVATION OF THE GENERAL CONSTANTS OF PARALLEL TRANSMISSION LINE

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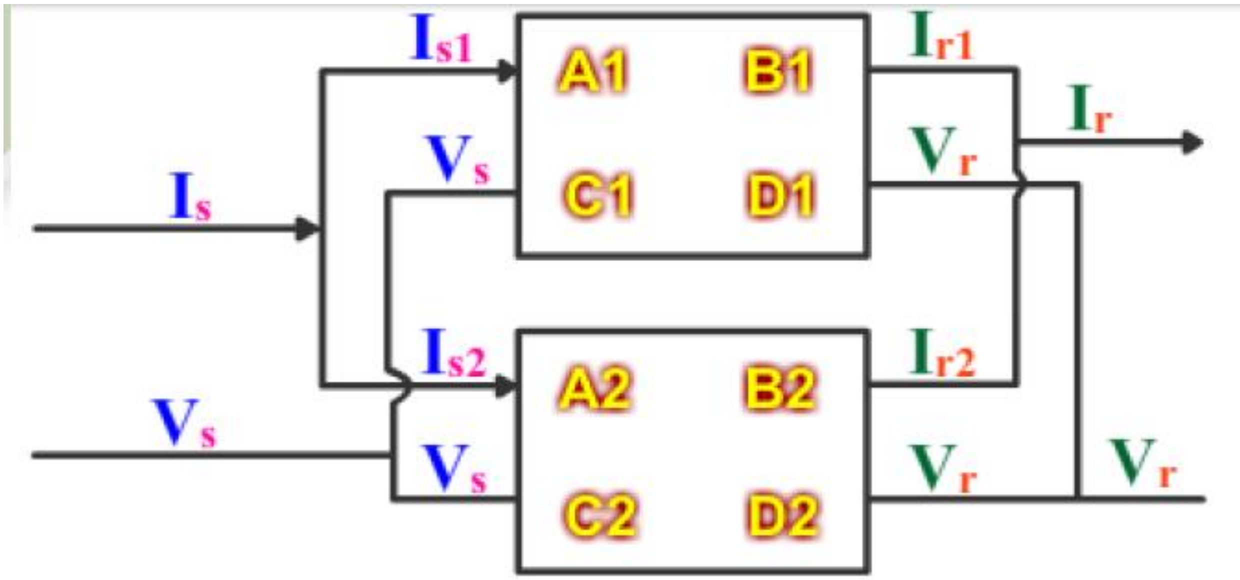

From the circuit diagram we conclude that :

$$V_s = A_1 V_r + B_1 I_{r_1} \qquad V_s = A_2 V_r + B_2 I_{r_2}$$

$$I_{s_1} = C_1 V_r + D_1 I_{r_1} \qquad I_{s_2} = C_2 V_r + D_2 I_{r_2}$$

$$I_s = I_{s_1} + I_{s_2} \qquad I_r = I_{r_1} + I_{r_2}$$

$$\therefore I_{r_2} = I_r - I_{r_1}$$

$$A_1 V_r + B_1 \, I_{r_1} = A_2 \, V_r + B_2 \left(I_r - I_{r_1} \right)$$

$$A_1 V_2 + B_1 I_{r_1} = A_2 V_r + B_2 I_r - B_2 I_{r_1}$$

$$I_{r_1} (B_1 + B_2) = B_2 I_r + V_r \, (A_2 - A_1)$$

$$\therefore \quad \boldsymbol{I_{r_1}} = \quad \left(\frac{\boldsymbol{B_2}}{\boldsymbol{B_1 + B_2}}\right)\boldsymbol{I_r} \quad + \left(\frac{\boldsymbol{A_2 - A_1}}{\boldsymbol{B_1 + B_2}}\right) \ \boldsymbol{V_r}$$

$$\because V_s = A_1V_r + B_1I_{r1} = A_1V_r \ + B_1 \ \left[\left(\frac{B_2}{B_1 + B_2}\right)I_r \quad + \left(\frac{A_2 - A_1}{B_1 + B_2}\right) \ V_r\right]$$

$$= A_1V_r + (\frac{B_1B_2}{B_1 + B_2})I_r + (\frac{B_1(A_2 - A_1)}{B_1 + B_2})\,V_r$$

$$\therefore V_s = V_r\left[\frac{B_1A_2 - \boldsymbol{B_1A_1}}{B_1 + B_2} + \frac{\boldsymbol{A_1B_1} + B_2A_2}{B_1 + B_2}\right] + \left(\frac{B_1B_2}{B_1 + B_2}\right)I_r$$

$$\therefore \quad V_s = \left[\frac{\boldsymbol{B_1A_2 + B_2A_1}}{\boldsymbol{B_1 + B_2}}\right]V_r + \left[\frac{\boldsymbol{B_1B_2}}{\boldsymbol{B_1 + B_2}}\right]I_r$$

$$= V_s = \boldsymbol{A}V_r + \boldsymbol{B}I_r$$

$$I_s = C_1V_r + D_1I_{r_1} + C_2V_r + D_2I_{r_2}$$

$$= \ V_r(\ C_1 + C_2) + D_1I_{r_1} + D_2I_r - D_2\ I_{r_1}$$

$$= V_r(C_1 + C_2) + D_1I_{r_1} + D_2I_r - D_2I_{r_1}$$

$$= V_r(\ C_1 + C_2) + D_2I_r + (D_1 - D_2\ )\left(\frac{B_2}{B_1 + B_2}\ \ I_r + \frac{A_2 - A_1}{B_1 + B_2}\ V_r\ \right)$$

$$= V_r(C_1 + C_2) + D_2I_r + \frac{B_2(D_1 - D_2)}{B_1 + B_2}\ I_r + \frac{(A_2 - A_1)(D_1 - D_2)}{B_1 + B_2}\ V_r$$

$$\therefore I_s = V_r\left[C_1 + C_2 + \frac{(A_2 - A_1)(D_1 - D_2)}{B_1 + B_2}\right] + I_r\left[\frac{B_2D_1 - B_2D_2 + B_2D_2 + B_1D_2}{B_1 + B_2}\right]$$

$$\therefore I_s = V_r\left[\boldsymbol{C_1} + \boldsymbol{C_2} + \frac{(\boldsymbol{A_2} - \boldsymbol{A_1})(\boldsymbol{D_1} - \boldsymbol{D_2})}{\boldsymbol{B_1} + \boldsymbol{B_2}}\right] + I_r\left[\frac{\boldsymbol{B_2D_1} + \boldsymbol{B_1D_2}}{\boldsymbol{B_1} + \boldsymbol{B_2}}\right]$$

$$I_s = \boldsymbol{C}V_r + \boldsymbol{D}I_r$$

$\therefore$ *The general constants of a parallel T.L. are*

$$\boldsymbol{A} = \left[\frac{\boldsymbol{B_1A_2} + \boldsymbol{B_2A_1}}{\boldsymbol{B_1} + \boldsymbol{B_2}}\right]$$

$$\boldsymbol{B} = \left[\frac{\boldsymbol{B_1B_2}}{\boldsymbol{B_1} + \boldsymbol{B_2}}\right]$$

$$\boldsymbol{C} = \left[\boldsymbol{C_1} + \boldsymbol{C_2} + \frac{(\boldsymbol{A_2} - \boldsymbol{A_1})(\boldsymbol{D_1} - \boldsymbol{D_2})}{\boldsymbol{B_1} + \boldsymbol{B_2}}\right]$$

$$\boldsymbol{D} = \left[\frac{\boldsymbol{B_2D_1} + \boldsymbol{B_1D_2}}{\boldsymbol{B_1} + \boldsymbol{B_2}}\right]$$